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# Gale–Nikaido–Debreu and Milgrom–Shannon: Communal interactions with endogenous community structures <sup>☆</sup>

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## Abstract

This paper examines Nash jurisdictional stability in a model with a continuum of agents whose characteristics are distributed over a unidimensional interval. Communal benefits and costs of each individual depend on her identity and the composition of the community which she belongs to. Since the framework is too general to yield an existence of Nash equilibrium, we introduce the essentiality of membership in one of the communities for all individuals. We highlight the Border Indifference Property (BIP), when all individuals located on a border between two adjacent jurisdictions are indifferent about joining either of them and show that BIP is a necessary condition for yielding a Nash equilibrium. We invoke the celebrated Gale–Nikaido–Debreu Lemma to guarantee the existence of a partition that satisfies BIP. We then proceed to demonstrate that BIP is not sufficient to yield a Nash equilibrium. The equilibrium existence under BIP is rescued when we use the Milgrom–Shannon monotone comparative statics conditions.

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